

Optimization study of fuzzy parametric uncertain system

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ABSTRACT

This paper deals with the analysis and design of the optimal robust controller for the fuzzy parametric uncertain system. An LTI system in which coefficients depends on parameters described by a fuzzy function is called as fuzzy parametric uncertain system. By optimal control design, we get control law and feedback gain matrix which can stabilize the system. The robust controller design is a difficult task so we go for the optimal control approach. The system can be converted into state space controllable canonical form with the α -cut property fuzzy. For optimal control design, we find control law and get the feedback gain matrix which can stabilize the system and optimizes the cost function. Stability analysis is done by using the Kharitonov theorem and Lyapunov-Popov method. The proposed method applied to a response of Continuous Stirred Tank Reactor (CSTR).

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1. INTRODUCTION

To design a proper controller for real time nonlinear system have many problems such as uncertainty, disturbance, unknown exact mathematical model etc. Uncertainty is either structure or Unstructured. Mostly the nonlinear system defined in terms of mathematical model doesn't have the exact parameter the calculated parameters are considered. So that type of model consists the parametric uncertainty. And because of that uncertain parameter, we have some information loss, which may be incomplete, unreliable. Also, the uncertainty affects the system response. So, we must design a controller such that it can deal with this type of uncertainty and remove the effects occurred because of that. For that, the fuzzy logic controller gives the best solution. In this paper, the design a controller which deal with parametric uncertainty with a fuzzy controller is given. That is the system with known mathematical model represent in terms of fuzzy membership function i.e. fuzzy parametric uncertain system. That means we consider some range of parameter variation. Then find the α -cut set for that fuzzy parametric uncertain system. From that we get the nominal system, then find the feedback gain matrix for that system which stabilize the system. We can design a control law for worst case condition i. e. for $\alpha = 0$ when the greatest uncertainty Considered. The stability is checked by Kharitonov theorem.

Robustness is the property of the system which deal with the variation in parameters in some range or bound [4]. In [5] this the method for fuzzy parametric uncertain system we discussed to design the robust controller with considering two examples. In first example, two uncertain parameters considered and in another example three uncertain parameters considered checked the response for that. The arithmetic calculation for α -cut set property of fuzzy are given in [1]. The stability of system is checked by Kharitonov theorem [3]. Because for this theorem we consider some range of parameter and in fuzzy parametric uncertain system uncertain

parameter considered in some range. In [2] the design of Popov-Lyapunov stability method is discussed. The design of paper is as follows, in section II represent the proposed methodology in that we discuss the α -cut for fuzzy parametric uncertain system. In section III we study the CSTR and apply the proposed method to for system and study the result. In section IV the stability of system by applying method is checked using kharitonov polynomial and Popov-Lypunov method [2]. In section V we end the paper.

2. PROPOSED METHODOLOGY

2.1. α -cut set

Fuzzy parametric uncertain system is the co-ordinates of system depend on parameters represented as fuzzy membership function. A parametric uncertainty is present when system represent in mathematical model form, which is not exactly known. Here, the uncertain parameters can be defined in-terms of fuzzy function $\tilde{p}_r \in p$ with membership function $\alpha = \mu(\tilde{p}_i) \in [0,1]$ as shown in Figure 1 The p is a universe of discourse for, \tilde{p}_i . The membership functions $\mu(\tilde{p}_r)$ are having single mode and decrease to the interval last limit. Thus, the fuzzy uncertain parameter \tilde{p}_r with α -cut is given by

$$\tilde{p}_r(\alpha_r) = [p_r^-(\alpha_r), p_r^+(\alpha_r)] \quad (1)$$

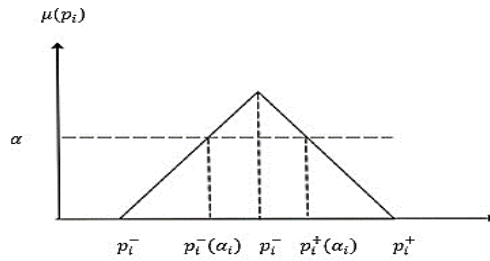


Figure 1. α cut for fuzzy uncertain parameter \tilde{p}_i

Where α_i is the membership height for the, \tilde{p}_r also $p_r^-(.)$ is a growing function and $p_r^+(.)$ is a reducing function. Therefore,

$$p_{r1}^-(0) = p_{r1}^- p_{r1}^+(0) = p_{r1}^+ p_{r1}^-(1) = p_{r1}^+ = p_{r1}^0 \quad (2)$$

The membership value α_i can be taken as the confidence degree form that we get the nominal system. The value $\alpha_r = 1$ indicates the precise knowledge.

$$p_{r1} = \ker(\tilde{p}_{r1}) = p_{r1}^0 \quad (3)$$

Whereas $\alpha_r = 0$ represents maximum uncertainty

$$p_{r1} = \text{supp}(\tilde{p}_{r1}) = [p_{r1}^-, p_{r1}^+] \quad (4)$$

2.2. Optimal controller design

Consider a plant with uncertainty:

$$\tilde{T}(s', \tilde{a}', \tilde{p}') = \frac{\tilde{a}'_{n-1} s'^{n-1} + \dots + \tilde{a}'_0}{s'^n + \tilde{p}'_{n-1} s'^{n-1} + \dots + \tilde{p}'_0} \quad (5)$$

Where \tilde{a}'_r and $\tilde{p}'_r, r = 0, 1, 2, \dots, n-1$, interpreted as fuzzy function. If a degree of confidence $\alpha \in [0, 1]$ in the coefficients is given, then in (5) can be interpreted as an interval system as:

$$\tilde{T}(s', \tilde{a}'(\alpha), \tilde{p}'(\alpha)) = \frac{\tilde{a}'_{n-1}(\alpha) s'^{n-1} + \dots + \tilde{a}'_0(\alpha)}{s'^n + \tilde{p}'_{n-1}(\alpha) s'^{n-1} + \dots + \tilde{p}'_0(\alpha)} \quad (6)$$

Where $\tilde{a}_r(\alpha)$ and $\tilde{a}_r(\alpha), r = 0, 1, 2, \dots, n-1$, interpreted as fuzzy function. The FPU system in (6) can be represent in state space controllable canonical form:

$$\dot{x}' = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\tilde{p}_0(\alpha) & -\tilde{p}_0(\alpha) & \cdots & -\tilde{p}_0(\alpha) \end{bmatrix} x' + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u'$$

$$y' = [-\tilde{a}_0(\alpha) \quad -\tilde{a}_1(\alpha) \quad \cdots \quad -\tilde{a}_{n-1}(\alpha)] x' \quad (7)$$

Where $\alpha \in [0, 1]$. The system in (7) represented as

$$\dot{x} = A'(\tilde{p}(\alpha))x + B'u$$

$$y' = C'(\tilde{a}(\alpha))x \quad (8)$$

Where \tilde{p}' is the parametric uncertainty vector in terms of α , and A, B, and C are the state space matrices. The uncertainty in state matrix (A') represented by (8) is balanced with B', if this uncertainty is in the bound of B'. Now a nominal value, $p_{nom} \in \tilde{p}(\alpha)$ such that $(A'(p_{nom}), B')$ is stable. Also, it is assumed that for any $\tilde{p}'(\alpha) \in p$ there exists a $1 \times n$ matrix $\Phi(\tilde{p}'(\alpha))$ so that, we can represent the uncertainty in State matrix (A') as

$$A'(\tilde{p}'(\alpha)) - A'(p_{nom}) = B'\Phi(\tilde{p}'(\alpha)) \quad (9)$$

Where $\Phi(\tilde{p}(\alpha))$ is bounded and $p_{nom} \in \tilde{p}(\alpha)$ is the nominal value of $\tilde{p}(\alpha)$

$$A'(\tilde{p}'(\alpha)) = A'(p_{nom}) + B'\Phi(\tilde{p}'(\alpha)) \quad (10)$$

From (10) the (8) becomes,

$$\dot{x}' = A'p_{nom}(\alpha)x' + B'\Phi(\tilde{p}'(\alpha))x' + B'u' \quad (11)$$

is stable for all $\tilde{p}(\alpha) \in p$ for all $\alpha \in [0, 1]$. Robust controller design for this problem is difficult. Hence optimal controller is designed for FPUS in (8). If the system is stable then only we can design LQR controller. The system in (9) is always controllable, hence it implies that pair $A'(\tilde{p}'(\alpha))B'$ is stable for each $\alpha \in [0, 1]$

LQR design

The problem can be solved using LQR controller which can be designed as:

For the nominal system correlate to each $\alpha \in [0, 1]$.

$$\dot{x}' = A(q_{nom})x' - Bu' \quad (12)$$

A control law, $u' = -k^*x'$ which reduces the cost functional

$$J'_1 = \int_0^\infty (x'^T F' x' + x'^T x' + u'^T R' u') dt \quad (13)$$

Where F' is an upper bound on the uncertainty $\Phi(\tilde{p}'(\alpha))^T \Phi(\tilde{p}'(\alpha))$, that is, for all $\tilde{p}(\alpha) \in p$

$$\Phi(\tilde{p}'(\alpha))^T \Phi(\tilde{p}'(\alpha)) \leq F' \quad (14)$$

The theorem in (11) can be used to design an optimal controller for FPUS in (8). When $\Phi(\tilde{p}'(\alpha))$ is enclosed the existence of the upper bound on F' is guaranteed. The cost function J can be written as

$$J'_1 = \int_0^\infty (x'^T Q' x' + u'^T R' u') dt \quad (15)$$

Where $Q' = [F' + I]$ and $R' = [1]$. Thus, if (14) satisfied then an optimal controller designed.

Error weighted matrix

Design a controller for $\alpha=0$ which consist of maximum ambiguity that controller stabilizes the all other system for different values of α . For $\alpha = 0$, the parametric ambiguity is described by $p_r \in [p_r^-, p_r^+]$, for $r = 0, 1, 2, \dots, n-1$. Calculate the error weighted matrix which satisfy the condition in (14) Consider the nominal value as,

$$p'_{nom} = [p_0^- \ p_1^- \ \dots \ p_{n-1}^-]$$

Take the nominal value,

$$p'_{nom} = [p_0^+ \ p_1^+ \ \dots \ p_{n-1}^+]$$

or any value in between $[p_r^- \ p_r^+]$ Then the nominal system of (8) is given by

$$\dot{x}' = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -p_0^- & -p_1^- & \dots & -p_{n-1}^- \end{bmatrix} x' + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u'$$

The uncertainty can be written as

$$\begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ p_0^- - \tilde{p}_0 & p_1^- - \tilde{p}_1 & \dots & p_{n-1}^- - \tilde{p}_{n-1} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} p_0^- - \tilde{p}_0 \\ p_1^- - \tilde{p}_1 \\ \vdots \\ p_{n-1}^- - \tilde{p}_{n-1} \end{bmatrix}$$

Hence the uncertainty satisfies the matching condition. The uncertainty is given by

$$\phi' = [p_0^- - \tilde{p}_0 \ p_1^- - \tilde{p}_1 \ p_{n-1}^- - \tilde{p}_{n-1}] \quad (18)$$

Which is bounded by

$$\phi'^T \phi' = F' \quad (19)$$

Using F' optimal controller is designed which reduces the cost function J .

3. CASE STUDY CSTR

A real-time experimental setup for highly nonlinear tank is constructed shown in Figure 2. DAC is used to interface CSTR with the Personal Computer (PC). The overall system consists of a tank, pump, Rotameter, RTD, an electro-pneumatic converter (I/P converter), a pneumatic control valve, an interfacing DAC module and a Personal Computer (PC). The differential pressure transmitter output is interfaced with the computer using DAC module in the RS-232 port of the PC. Figure 3 shows the block diagram of a CSTR tank interfaced with PC. The pneumatic control valve uses air as an input and adjusts the flow of the water pumped to the CSTR jacket from cold water tank. This flow maintains the temperature inside the tank at the desired value. The temperature of the liquid inside the tank is measured with the help of RTD and is transmitted in the form of (4-20) mA to the interfacing DAC module with the help of temperature transmitter to the Personal Computer (PC).



Figure 2. Experimental setup

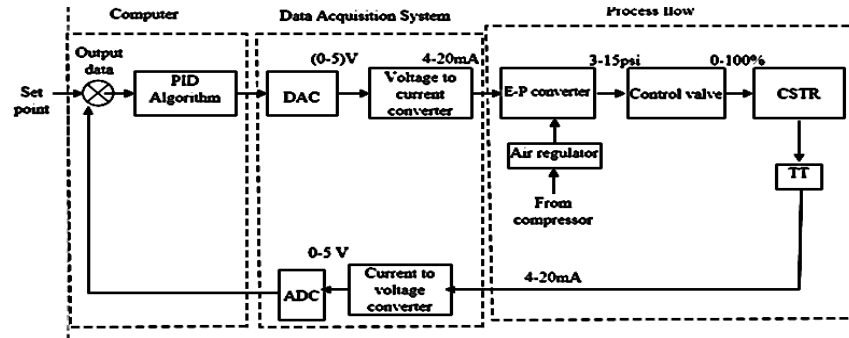


Figure 3. Block diagram of CSTR

In Figure 3 the block diagram for CSTR gives in that we see the three regions first computer in that we design a controller which connect to the system through the data acquisition system, third is process flow. After calculating the control algorithm in the PC, required control signal in the form of current signal (4-20) mA is transmitted to the I/P converter, which passes the air signal to .operate by this signal to produce the required flow of water in and out of the tank.

Calculation of TF:

For calculating Transfer function of CSTR cooling process, the step response is taken into consideration. The transfer function is calculated by using process reaction curve. The process has very large dead time and is highly damped. Therefore, the step response can be fitted into a simple first-order model with dead-time.

$$G'(s) = \frac{A'e^{-\theta's}}{\tau's + 1}$$

Where, A=Process gain, θ' =Dead-time, τ' =Time constant Therefore, the transfer function of the process is given by

$$G'(s) = \frac{0.12e^{-2s}}{3s + 1}$$

The Transfer Function of Valve is

$$G'_v(s) = \frac{0.112}{0.8s + 1}$$

By using Pade's approximation the second order transfer function is calculated as

$$G'(s) = \frac{-0.12s + 0.12}{3s^2 + 4s + 1}$$

The state space matrices are given as

$$\begin{aligned} A'_1 &= \begin{bmatrix} -1.33 & -0.667 \\ 0.5 & 0 \end{bmatrix} & B'_1 &= \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} \\ C'_1 &= [-0.16 \quad 0.32] & D'_1 &= [0] \end{aligned}$$

Simulink design: `

3.1. Proposed Technique

System transfer function

$$G'_{sys}(s) = \frac{0.0056}{s^2 + 1.583s + 0.417}$$

Consider two uncertain parameters which represent in fuzzy no. as first $a=\text{tri}(0.03\ 0.05\ 0.07)$ and second is $b=\text{tri}(1.2\ 1.5\ 1.7)$ and $c=\text{tri}(0.2\ 0.4\ 0.6)$. The α cut for this is $a=[(0.02\alpha+0.03), (0.07-0.02\alpha)]$ $b=[(0.2\alpha+1.2), (1.7-0.2\alpha)]$, and $c=[(0.2\alpha+0.2), (0.6-0.2\alpha)]$. For different values of α the values of a, b, c are given in table

Table 1. α -cut set				
$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$
[0.03 0.07]	[0.035 0.065]	[0.04 0.06]	[0.045 0.055]	[0.05 0.05]
[1.2 1.7]	[1.25 1.75]	[1.3 1.6]	[1.35 1.55]	[1.4 1.4]
[0.2 0.6]	[0.25 0.55]	[0.3 0.5]	[0.35 0.45]	[0.4 0.4]

Feedback gain matrix: $K' = [0.8954\ 0.9168]$

The closed loop characteristic Equation for $\alpha = 0$:

$$[1.1168, 1.5168] + [2.095, 2.59]s + s^2 = 0$$

Kharitonov polynomials are:

$$K'_1 = 1.5168 + 2.59s + s^2$$

$$K'_2 = 1.5168 + 2.095s + s^2$$

$$K'_3 = 1.1168 + 2.095s + s^2$$

$$K'_4 = 1.1168 + 2.59s + s^2$$

Step Response for different values of $\alpha \in [0, 1]$

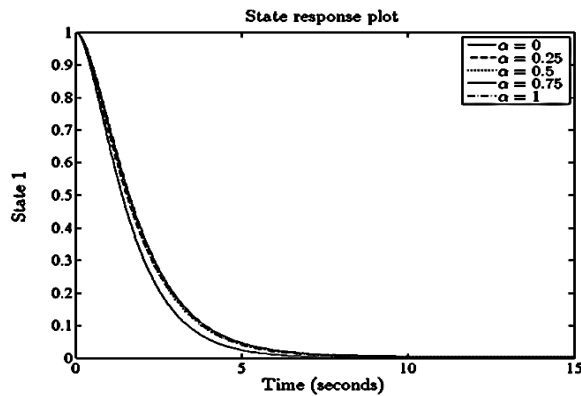


Figure 4. Step Response of states for different values of α

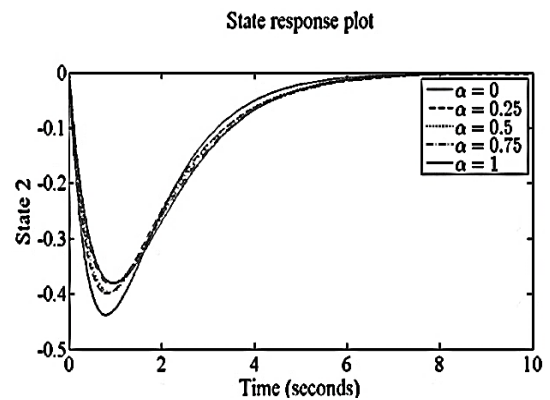


Figure 5. Step Response of states for different values of α

In Figure 4 and Figure 5 shows the state response for different values of α . In that, the response for $\alpha=0$ gives the stable response for other values α . In Figure 6 the bode plot for interval plant is shown.

Bode plot for interval plant:

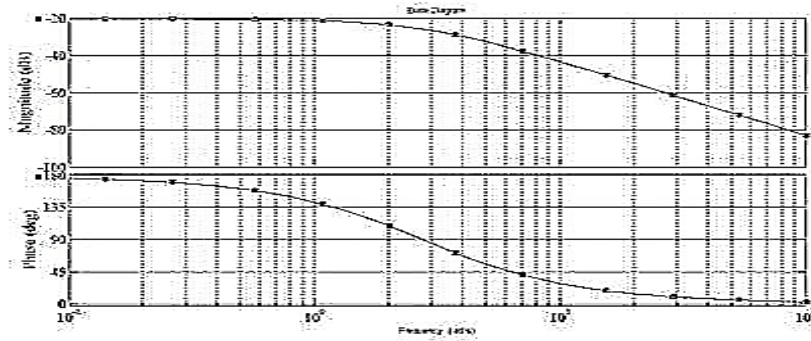


Figure 6. Bode plot

Algorithms:

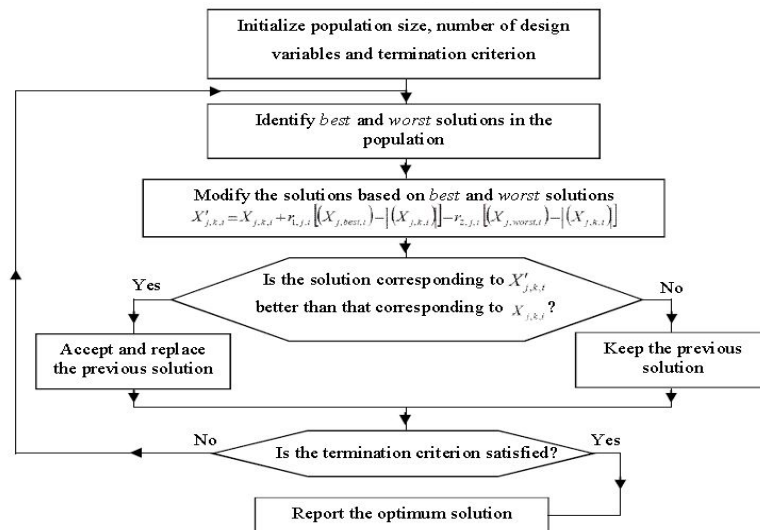


Figure 7. Flowchart for JAYA algorithm

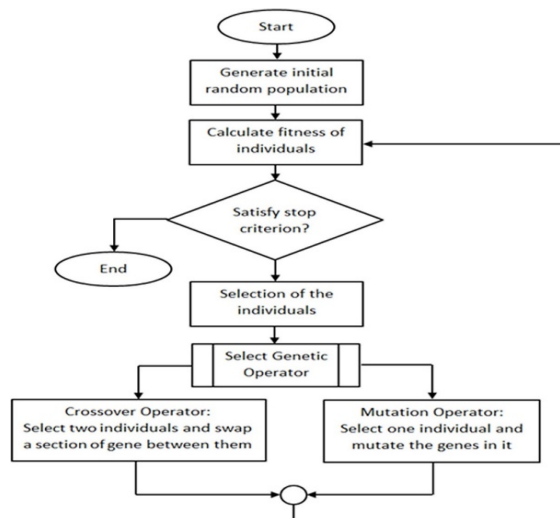


Figure 8. Flowchart for GA algorithm

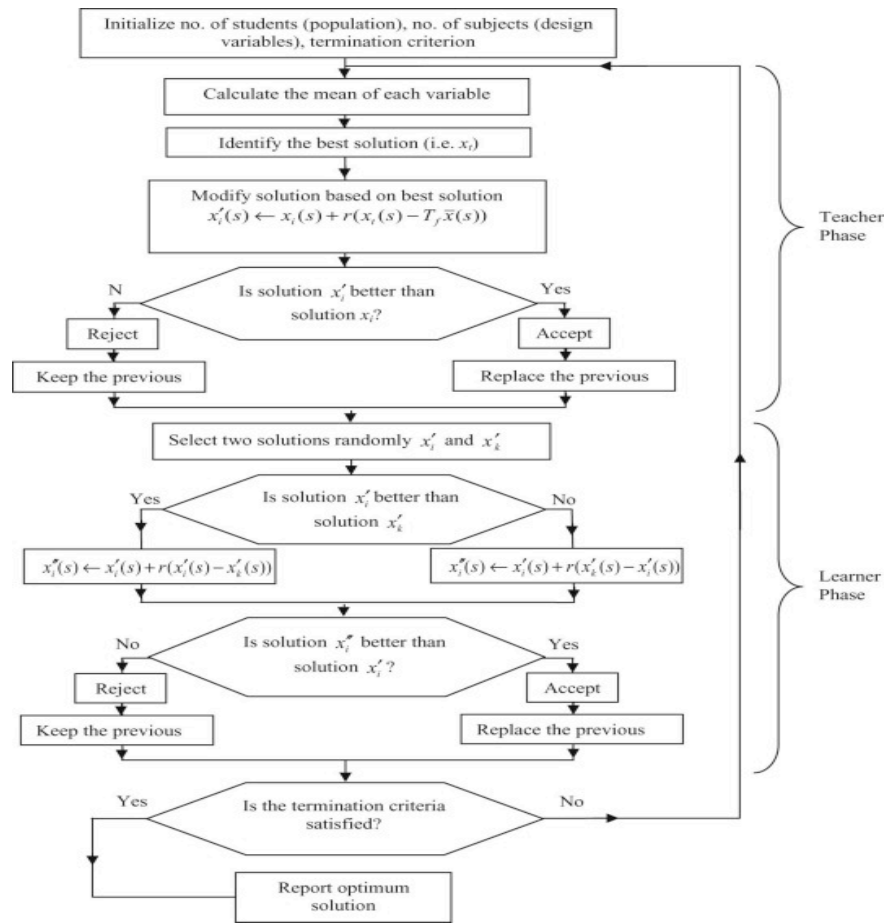


Figure 9. Flowchart for TLBO algorithm

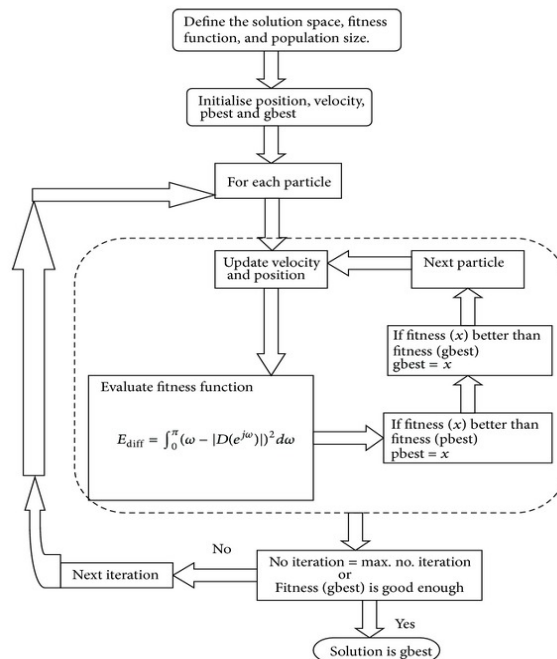


Figure 10. Flowchart for PSO algorithm

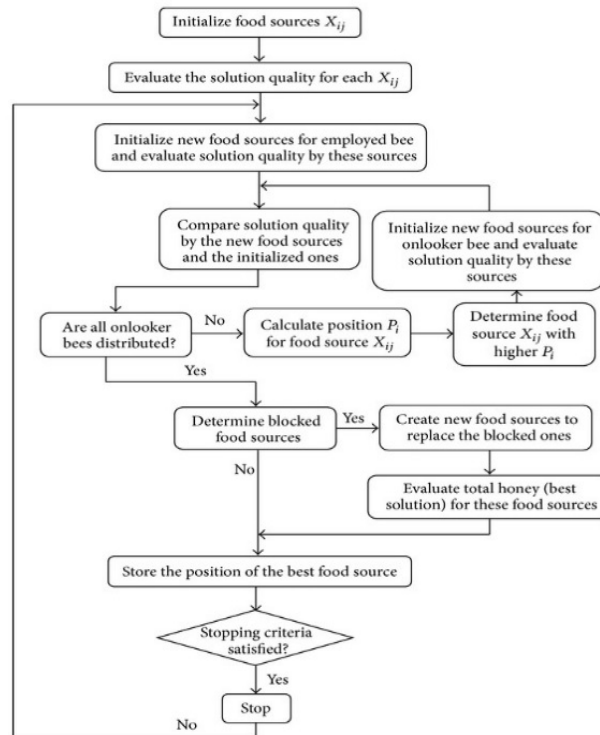


Figure 11. Flowchart for ABC algorithm

To avoid membership function as too redundant or too separate we use this algorithms for optimization. Fuzzy JAYA gives best result among all this algorithms namely GA, TLBO, ABC, PSO.

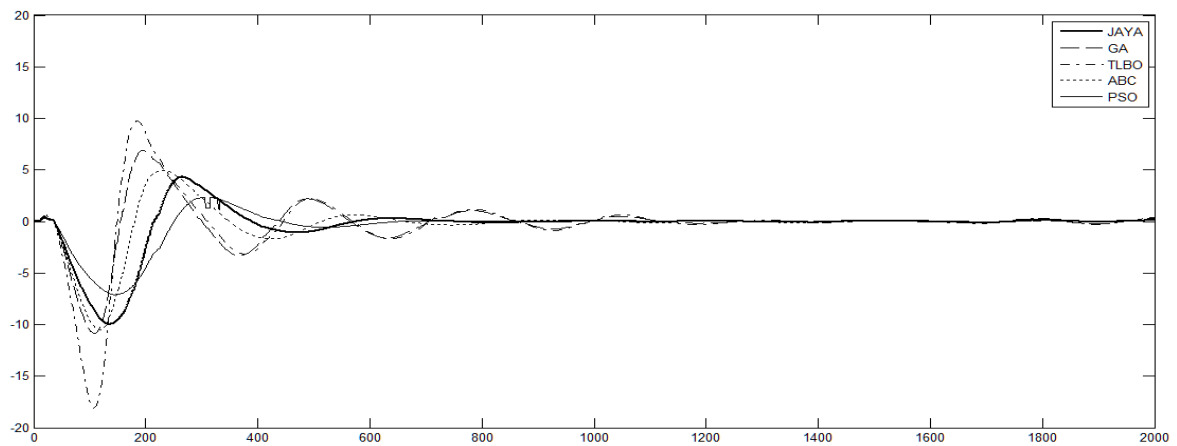


Figure 12. comparative results by using different algorithms

Table 2. Comparative Parameters with different algorithms

Algorithms	Rise Time	Settling Time	Overshoot	Peak	ISE
JAYA	1.9947	4.5821e+03	1.0207e+04	18.0752	381.2
GA	0.3289	5.0521e+03	2.8753e+04	9.9626	415.7
TLBO	1.0061	5.1323e+03	1.5342e+05	10.8502	475.7
ABC	1.4707	5.5821e+03	1.8207e+04	18.0752	475.7

PSO	1.4707	5.5691e+03	1.8207e+04	10.5219	345.7
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3.2. Hardware Result

For this, we have considered the error as 0-5 i.e. the difference between the setpoint and the actual value is 0-5 and control action is taken between 0-100.

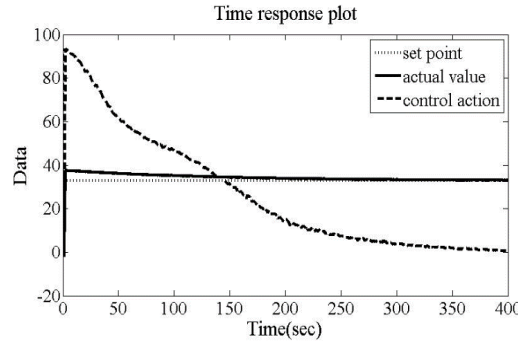


Figure 13. Hardware Result

The figure. 9 shows the response for the real-time system. In this, we see that the set point is tracked by the actual temperature value. Also at the same time control action is also plotted.

4. STABILITY ANALYSIS

4.1. Kharitonov Theorem

Consider a family n of real rational polynomials

$$\Delta(s) = \Delta_1 s + \Delta_2 s^2 + \dots + \Delta_{n-1} s^{n-1} + \Delta_n s^n \quad (20)$$

This polynomial family is called the family of uncertain or interval polynomial.

$$x_i \leq \Delta_i \leq y_i, \text{ where, } i=0, 1, 2, \dots, n.$$

In accordance with Kharitonov theorem every polynomial in the family $\Delta(s)$ is Hurwitz if and only if the following four extreme polynomials are Hurwitz [3].

$$K'_1 = x_0 + x_1 s + y_2 s^2 + y_3 s^3 +$$

$$K'_2 = y_0 + y_1 s + x_2 s^2 + x_3 s^3 +$$

$$K'_3 = y_0 + x_1 s + x_2 s^2 + y_3 s^3 +$$

$$K'_4 = x_0 + y_1 s + y_2 s^2 + x_3 s^3 +$$

To apply this theorem, we need consider some range and for fuzzy parametric uncertain system we consider uncertain parameter in the form some range.

4.2. Popov-Lyapunov Method

The stability of fuzzy control system is checked by Popov-Lyapunov approach [2]. In order to do this, we transform the fuzzy system into Lure system with uncertainty. After that Lyapunov direct method is used to guarantee the stability than the robustness measurement which gives the bound on allowable uncertainty. The allowable bounds can help us to estimate the robust stability of fuzzy control system. The dynamic Equation of the system is

$$\dot{x}' = \begin{bmatrix} 0 & 1 \\ -0.417 & -1.583 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u'$$

$$y' = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

From (21) we know the A_0, B_0, C_0 . Now, first consider fuzzy control system,

$$e' = r' - y'$$

Where, r' is reference input and u' is controller output. For this system. K_1, K_2 And. K_u are the gain values which will be 0.8954, 0.9168 and 1 respectively. And K_1, K_2 , are the feedback gain values.

The system can be representing in the form of,

$$A' = A'_0 - K'_u K'_{min} b'_0 C'_1, \quad b' = -K'_u b'_0 \quad [23]$$

$C' = -C'_1, \dot{x} = A'x - b'_0 \sigma = -C'^T_1 x$ the values of A', b' and c' are,

$$A'_1 = \begin{bmatrix} 0 & 1 \\ -2.59 & -1.5196 \end{bmatrix}, b' = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C' = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad [24]$$

The transfer function can be obtained as

$$G' = C(sI - A)^{-1} b \quad [25]$$

This is the Lur'e perturbed system. The system defined in (25) satisfies the following condition then the system is asymptotically stable

1. The nonlinearity σ always belongs to the sector $[0, K_1^-]$ where K_1^- is a positive number.
2. The system matrix A_1 is Hurwitz ($G(s)$ is stable), and there exists a scalar $r > 0$ such that $-1/r \leq \lambda_i$, where λ_i is an eigenvalue of A_1 , and

$$1/K_1^- + R'e [1 + j\omega r] G'(j\omega) > 0, \forall \omega \in R' \quad [26]$$

3. Let $\gamma = \frac{1}{2rA^T c} + c, \gamma = rC^T b + \frac{1}{K_1^-} \gamma = rc^T$ (27) where r is chosen such that $\gamma \geq 0$. Where W is symmetric positive-definite matrix, there exists $\alpha > 0$, a vector q , P is symmetric positive-definite matrix and W_0 , and $\delta > 0$ satisfying

$$\begin{aligned} A'^T P' + P' A' &= -q' q'^T - \epsilon W' P' b' - v' \sqrt{\gamma'} q' \\ &= \epsilon W' \\ &= \epsilon W' \theta' + \delta' I' \end{aligned} \quad [28]$$

The robust stability is found by satisfying theorems 1, 2 and 3

$$G'(s) = \frac{1}{s^2 + 1.519s + 2.59}$$

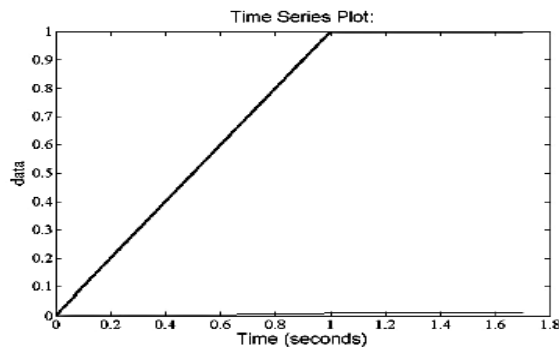


Figure 14. Popov plot

In Figure 10 the Popov plot is shown. From this plot, we can find the slope of the line which is used for the further calculations. From the Popov plot for $G(j\omega)$, we get the value or $r=0.5$.

$$V \begin{bmatrix} -1.1070 \\ 0.1503 \end{bmatrix} \text{ and}$$

$$\gamma = 5.0198$$

$$\text{Let } \varepsilon W' = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix}$$

After solving the algebraic Riccati (28) get the P matrix as

$$P' = \begin{bmatrix} 4.4455 & 0.5796 \\ 0.5796 & 1.1618 \end{bmatrix} \text{ and } \beta=0.2591 \quad [29]$$

The P matrix is positive definite, so the system is stable.

5. CONCLUSION

The controller is designed for FPUS using the α -cut property of fuzzy set. Controller designed for a critical condition that is for maximum uncertainty interval and that will stabilize the other interval of uncertainty. This technique is applied to continuous stirred tank reactor and study the responses. Also, the proposed technique is applied with a fuzzy optimized membership function using Jaya algorithm. The stability of the system is checked by Kharitonov polynomials and Popov-Lyapunov stability theorem.

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